American Regions Math League

Some Mathematical Ideas
Whose Understanding Will Be Assumed For This Contest

1. The word *"compute"* will always call for an answer in simplest form. Thus final answers like $\frac{6}{4}$, 5 + 2, 2^5 and $2\sin 30°$, for example, would not be satisfactory. In cases where there is question as to what is "most simplified", alternate answers may be accepted (example: $\frac{3}{2}$ and $1\frac{1}{2}$ are both acceptable). Judges' decision is final.
2. When an answer is called for as an *ordered pair, (a,b),* it must be given in precisely that form, including the parentheses and the comma. The same applies for other choices of letters and for ordered n-tuples.
3. The *sides opposite vertices A, B, and C of triangle ABC* will be represented by the lower case *letters a, b,* and *c,* respectively. Depending on the context, *A* can represent the vertex, or the angle, or the measure of the angle, and *a* can represent the side or its length. A similar convention holds for other choices of letters representing a triangle. If a quadrilateral is named *ARML*, it is understood that the vertices *A, R, M,* and *L* occur in this order around the polygon (either clockwise or counterclockwise). This convention holds for other choices of letters and for the naming of polygons in general. When referring to polygons (including triangles), we are referring to non-degenerate ones.
4. The *Fibonacci sequence* is the sequence 1, 1, 2, 3, 5, 8, 13, . . . , where each term is the sum of the two previous terms. More formally, $F_1 = F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$.
5. The *Greatest Integer Function,* symbolized by brackets, is defined as follows: if *n* is an integer and if $n \leq x < n+1$, then [x] = n. Since brackets are often used simply as parentheses are, any problem using brackets to represent the *Greatest Integer Function* will clearly say so.
6. *Logs* are base 10 unless otherwise indicated; the use of log *x* also implies that *x* is positive. In general, when bases are not indicated, numbers referred to are in base 10. If another base is being used, that base will *usually* be written as a subscript. Examples: $\log_4 64$ [which equals 3]; or $102.13_{(4)}$ [which equals $18\frac{7}{16}$].
7. The letter *i* will always be used for $\sqrt{-1}$.
8. Some *symbols of Combinations and Permutations*: $\binom{n}{r} = {}_nC_r = \frac{n!}{(n-r)!(r)!}$; this is the number of combinations of n things taken r at a time. ${}_nP_r = \frac{n!}{(n-r)!}$; this is the number of permutations of n things taken r at a time. Note: 0! = 1.
9. The capital *A* that begins the expressions Arcsin *x*, Arccos *x*, and Arctan *x* calls for the principal values of these inverse trigonometric functions. The ranges are as follows: $-\frac{\pi}{2} \leq \text{Arc}\sin x \leq \frac{\pi}{2}$, $0 \leq \text{Arc}\cos x \leq \pi$, $-\frac{\pi}{2} \leq \text{Arc}\tan x \leq \frac{\pi}{2}$.
   [Degrees should be used instead of radians if the problem uses degrees.]
10. *Lattice points* are points on a grid, both of whose coordinates are integers.
11. *Divisors* (or factors) of an integer refers to positive integer divisors only. *Proper divisors* of an integer are divisors that are less than the integer itself.
12. The designation *primes* refers to positive primes only.
13. Sometimes problems refer to the *digits* of a number; in that case, those digits are usually underlined. Examples: "Let N = $\underline{7}\ \underline{7}\ \underline{7}$ . . . $\underline{7}\ \underline{7}$, where the digit 7 occurs 100 times"; or "Find the missing digits A and B if K = $\underline{A}\ \underline{2}\ \underline{5}\ \underline{B}$ and K is a multiple of 72." [The number K is *not* to be interpreted as the product of A, 2, 5, and B.]
14. If a *diagram* is given with a problem, it is not necessarily drawn to scale.
15. It is often helpful to have a basic knowledge of elementary number theory (including modular arithmetic) and of analytic geometry (including the conic sections) for these contests.
16. The *greatest lower bound* of a set is the largest number which is less than or equal to all the elements of the set. Thus 2 is the greatest lower bound for both $\{x : 2<x\}$ and $\{x : 2 \leq x\}$. The *least upper bound* of a set is the smallest number which is greater than or equal to all elements of the set. Thus 3 is the least upper bound for both $\{x : x<3\}$ and $\{x : x \leq 3\}$

[Revised: March 1999]